1. Find the other five trigonometric functions of $\theta$ using the given information.
$\tan \theta=\frac{5}{12} ; \sin \theta>0$
2. How far are you from the base of a tree 100 feet tall if the angle of elevation to the top of the tree is $30^{\circ}$ ?
3. Graph exactly one period of each of the following functions.
A. $y=2 \sin x+1$
B. $y=\tan x$
C. $y=\cos 2 x-2$
4. Prove the following identities by converting one side into the other.
A. $\sec x+\tan x=\frac{\csc x+1}{\csc x \cos x}$
B. $2-\sin ^{2} \theta=1+\cos ^{2} \theta$
C. $\frac{1}{1+\cos y}+\frac{1}{1-\cos y}=2 \csc ^{2} y$
5. Find the exact value of each of the following expressions.
A. $\operatorname{Arcsin} \frac{1}{2}$
B. $\operatorname{Cos}^{-1} \frac{\sqrt{2}}{2}$
C. $\cos ^{2} 15^{\circ}+\sin ^{2} 15^{\circ}$
D. $\csc 60^{\circ}$
E. $\tan \left[\sec ^{-1}(-3)\right]$
F. $\cot \frac{\pi}{4}$
6. Solve the following equations for all $x$ such that $0 \leq x<2 \pi$.
A. $2 \cos x \sin x=\sin x$
B. $\sec ^{2} x-\sec x-2=0$
C. $\cos ^{2} x=\frac{1}{2}$
7. Solve the following triangles for the unknown quantity.
A.

B.

2
8. For the polar equation $r=2-3 \sin \theta$, find the missing coordinate of the following points so that the resulting point will be in the graph of this equation.
A. $\left(?, \frac{5 \pi}{4}\right)$
B. $(2, ?)$
C. $\left(?, \frac{3 \pi}{2}\right)$
9. Change the polar coordinates $\left(-3, \frac{5 \pi}{4}\right)$ to rectangular coordinates.
10. Find an equation of the line that passes through the point $(1,-2)$ and that is perpendicular to the line $2 x+4 y=1$.
11. Find the inverse function of $f(x)=\frac{2 x-5}{x-3}$, if it exists.
12. Find all three zeros (or roots) of the polynomial $P(x)=x^{3}+x$.
13. Solve for $x: \log (28+x)=\log (2-x)+\log (2-x)$.
14. Solve for $x: 7^{3 x+1}=100$.
15. Find all horizontal and vertical asymptotes, if any, of $r(x)=\frac{5 x+21}{x^{2}+10 x+25}$.
16. If $f(x)=3 x^{2}+1$, simplify the following expression: $\frac{f(x+h)-f(x)}{h}$.
17. Identify the conic section given by the following equation and find its center:

$$
x^{2}+y^{2}-4 x+10 y+25=0
$$

9. Find the rectangular coordinates for the point whose polar coordinates are $(\sqrt{2},-\pi / 4)$.
10. Find functions $f(x)$ and $g(x)$ so that $f(g(x))=(2 x+1)^{3}$.
11. A function $f$ is given, and the indicated transformations are applied to its graph, in the given order. Write the equation for the final transformed function $g(x)$. $f(x)=x^{2}$; shift 2 units to the left and reflect in the $x$-axis.
12. Find the equation of the quadratic function $f(x)$ whose graph has vertex $(3,5)$ and $y$-intercept 23 .
13. Find the value of $b$ if $\log _{3} b=-2$.
14. Find the sum of the first five terms of the arithmetic sequence whose first term is 25 and common difference is -2 .
15. Find all possible rational zeros (or roots) of the polynomial $f(x)=2 x^{3}-7 x^{2}+10 x-6$.
16. Find the formula for a polynomial $f$ of degree 3 with integer coefficients such that $f(1)=-2$ and both $1-i$ and 3 are zeros (or roots) of the polynomial.
17. Find the value of $\tan ^{-1}(-1)$.
18. Find polar coordinates for the point whose rectangular coordinates are $(0,-3)$.
19. Find the sum of the first four terms of the geometric sequence whose first term is 3 and common ratio is 2 .
20. Find the equation of the circle that has a diameter with endpoints at $(-3,-2)$ and $(5,4)$.
21. Write the complex number $-1+i \sqrt{3}$ in the polar form $r(\cos \theta+i \cdot \sin \theta)$.
22. Use DeMoivre's Theorem to compute $(-1+i \sqrt{3})^{12}$.
23. Find the two foci of the ellipse with equation

$$
\frac{x^{2}}{36}+\frac{y^{2}}{25}=1
$$

24. Evaluate the expression

$$
\sum_{k=0}^{3}\left(k^{3}+2 k\right)
$$

25. A sequence is defined recursively by $F_{n}=F_{n-1}+F_{n-2}$. If $F_{0}=0$ and $F_{1}=1$, find $F_{6}$.

In problems 26-31, sketch the graph of the given function or equation.
26. $\frac{x^{2}}{4}+\frac{y^{2}}{16}=1$
27. $f(x)=\frac{2 x-3}{x-2}$
28. $y=e^{x}+2$
29. $y=\left\{\begin{aligned} 3, & \text { if } x \leq 0 \\ -x^{2}-2, & \text { if } x>0\end{aligned}\right.$
30. $y=x^{3}+2 x^{2}-5 x-3$
31. $y=\log _{1 / 2}(x+3)$
32. A polynomial of degree 4 can have at most how many $x$-intercepts?
33. A polynomial of degree 4 can have at most how many local extrema?

## ANSWERS FOR SAMPLE PLACEMENT TEST TO BYPASS MATH 1505 (PART I - TRIGONOMETRY REVIEW)

1. $\sin \theta=\frac{5}{13} \quad \csc \theta=\frac{13}{5} \quad \cos \theta=\frac{12}{13} \quad \sec \theta=\frac{13}{12} \quad \tan \theta=\frac{5}{12} \quad \cot \theta=\frac{12}{5}$
2. Distance $=100 \sqrt{3}$
3. 

A.

C.

B.

5. A. $\operatorname{Arcsin} \frac{1}{2}=30^{\circ}$ or $\frac{\pi}{6}$
B. $\operatorname{Cos}^{-1} \frac{\sqrt{2}}{2}=45^{\circ}$ or $\frac{\pi}{4}$
C. $\cos ^{2} 15^{\circ}+\sin ^{2} 15^{\circ}=1$
D. $\csc 60^{\circ}=\frac{2 \sqrt{3}}{3}$
E. $\tan \left[\sec ^{-1}(-3)\right]=-2 \sqrt{2}$
F. $\quad \cot \frac{\pi}{4}=1$
6.
A. $x=0^{\circ}, 60^{\circ}, 180^{\circ}, 300^{\circ}$ or $x=0, \frac{\pi}{3}, \pi, \frac{5 \pi}{3}$
B. $x=60^{\circ}, 180^{\circ}, 300^{\circ}$ or $x=\frac{\pi}{3}, \pi, \frac{5 \pi}{3}$
C. $x=45^{\circ}, 135^{\circ}, 225^{\circ}, 315^{\circ}$ or $x=\frac{\pi}{4}, \frac{3 \pi}{4}, \frac{5 \pi}{4}, \frac{7 \pi}{4}$
7. A. $x=5 \sqrt{2}$
B. $x=\sqrt{19}$
8.
A. $x=\frac{4+3 \sqrt{2}}{2}$
B. $x=0, \pm \pi, \pm 2 \pi, \pm 3 \pi, \cdots$
C. 5
9. $\left(\frac{3 \sqrt{2}}{2}, \frac{3 \sqrt{2}}{2}\right)$

## ANSWERS FOR SAMPLE PLACEMENT TEST TO BYPASS MATH 1505 (PART II - PRECALCULUS REVIEW)

| $1.2 x-y=4$ | 2. $f^{-1}(x)=\frac{3 x-5}{x-2}$ | 3. $x=0, x= \pm i$ |
| :---: | :---: | :---: |
| 4. $x=-3$ | 5. $x=\frac{2}{3 \log 7}-\frac{1}{3}$ | 6. Horizontal asymptote is $y=0$. Vertical asymptote is $x=-5$. |
| 7. $6 x+3 h$ | 8. A circle with center $(2,-5) .$ | 9. $(1,-1)$ |
| $\text { 10. } \begin{aligned} f(x) & =x^{3} \\ g(x) & =2 x+1 \end{aligned}$ | 11. $g(x)=-(x+2)^{2}$ | 12. $f(x)=2 x^{2}-12 x+23$ |
| 13. $1 / 9$ | 14. 105 | 15. $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$ |
| 16. $f(x)=x^{3}-5 x^{2}+8 x-6$ | 17. $-\frac{\pi}{4}$ | 18. $\left(3, \frac{3 \pi}{2}\right)$ |
| 19. 45 | 20. $(x-1)^{2}+(y-1)^{2}=25$ | 21. $2\left(\cos \frac{2 \pi}{3}+i \cdot \sin \frac{2 \pi}{3}\right)$ |
| 22. $2^{12}$ | 23. $(\sqrt{11}, 0),(-\sqrt{11}, 0)$ | 24. 48 |
| 25. 8 | 26. | 27. |
| 28. | 29. | 30. |
| 31. | 32. 4 | 33. 3 |

